# Notes Toward a Formal
# Model of Common Law

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This paper outlines steps towards a formal set theoretic model of common law. It is the result of an investigation begun out of curiosity alone; could the basic notions of common law be captured in the kind of set theoretic analysis that has been used so productively by Montague,[1] Cresswell,[2] and others in the semantics of natural language? If so, our understanding of common law and the possibility of its mechanization would be considerably advanced. If not, the reasons should prove illuminating. Either way the investigation of the formal properties of the relations involved, in particular of the precedent relation, should be useful. It turned out that it is possible to go quite a long way toward such a formalization, but that in two critical respects the model is not deterministic.[3] A formal model of common law decision making, however, should isolate its indeterminacies and precisely delineate the roles played out by these indeterminacies.

The characteristic question that the common law judge must answer is whether the factual situation before her falls under some particular legal predicate.[4] That is, is this situation a battery, breach of contract, etc.? By the time a judge is faced with this intrinsically legal question the facts of the case, the events on which the dispute is founded, have already been determined by jury (or by judge in a bench trial).[5] Of course actual events are neither before the court nor determined by it; rather, a linguistic representation of them is settled as accurate.[6] Accordingly, the facts of a case are comprised of a set of propositions, each determining some particular

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1. R. MONTAGUE, FORMAL PHILOSOPHY 188-270 (R. Thomason ed. 1974).

2. M. CRESSWELL, LOGICS AND LANGUAGES (1973).

3. This is hardly surprising. One should not expect stable and precise expressions of substantive content in a formal model of common law; on the contrary, a model that was substantively deterministic would be *ipso facto* inadequate.

4. Statutes and constitutions also generate legal predicates and delimit their application. This paper, however, focuses on common law predicates, a rather messier and less determinate lot. There is nothing magical about common law predicates. They are merely words that have proved convenient for the collection and classification of prior decisions and which have thus worked their way into the language of common law.

5. Of course the division between the functions of jury and judge are not precisely demarcated; even in his formal role the judge has a great influence over what facts a jury can find, for example, in controlling discovery, in deciding the admissibility of evidence, and in his instructions to the jury. For present purposes the simplification used here is justifiable for model building, but is more appropriate to the appellate level.

6. Sinclair, *The Semantics of Common Law Predicates,* 61 IND. L.J. 373, 378-79 (1985).

fact. The judge must decide whether the conjunction of these propositions is an instance of the legal predicate in question.[7]

The common law, unlike statutory law, contains no codified rule for the ascription of a legal predicate to a set of facts. It does not "admit the possibility of a court, however elevated, reaching a final, authoritative statement of what the law is in a general abstract sense."[8] At common law, a judge must decide for each case whether or not it instantiates a particular legal predicate. This decision is essentially a stipulation; the judge stipulates that this particular set of facts does or does not instantiate the legal predicate.[9] But, just because the decision is a stipulation, it does not follow that the judge is free of legal constraint in making it.[10] The doctrine of *stare decisis* is the central feature of common law that constrains judges and lends a crucial element of stability and predictability to the common law. Accounting for the role of *stare decisis* in judicial decision making is therefore the central problem in explaining common law.

No two cases are identical in all respects. The facts presented to the court in any given instance are unique, being at least spatio-temporally distinct and usually involving different persons.[11] All cases which have not previously been decided are therefore distinguishable on some grounds from all decided cases. Conversely, there is some respect in which any two cases are indistinguishable. To say that some distinctions are trivial is merely to beg the question of triviality. Only under some criterion of similarity can any two cases be held similar or distinguished.

> There is therefore a certain complexity in the structure of the idea of justice. We may say that it consists of two parts: a uniform or constant feature, summarized in the precept 'Treat like cases alike' and a shifting or varying criterion used in determining when, for a given purpose, cases are alike or different.[12]

It is the second feature, the criterion of similarity and difference, that is the most crucial determinant of the precedent relation, *stare decisis*.

For every established legal predicate, the judge has access to a number

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7. There is neither an authoritatively determined set of common law predicates nor any determinate method for discovering them. One could, however, make a more or less serviceable list to which a judge would be free to add as she found necessary. An example of a possible addition is the proposed tort of "satiric appropriation." Dorsen, *Satiric Appropriation and the Law of Libel, Trademark, and Copyright: Remedies Without Wrongs,* 65 B.U.L. REV. 923 (1985).

8. Simpson, *The Common Law and Legal Theory,* in OXFORD ESSAYS IN JURISPRUDENCE 77, 90 (A.W.B. Simpson 2d ed. 1973).

9. *See* Sinclair, *supra* note 6, at 387-89.

10. Indeed one of the most significant and contentious theses at large in jurisprudence today is that it is completely legally constrained. *See* Dworkin, *The Model of Rules,* 35 U. CHI. L. REV. 14 (1967); Raz, *Legal Principles and the Limits of Law,* 81 YALE L.J. 823 (1972). This dispute does not affect the formally stipulative nature of the judge's decision.

11. Cases of *res judicata* are the obvious exception and are the limiting situation for the analysis presented here. *See* Sinclair, *supra* note 6, at 390.

12. H.L.A. HART, THE CONCEPT OF LAW 156 (1961).

of reported judicial opinions presenting a set of facts, a holding that those facts do or do not instantiate that predicate, and a justification. If, within that set of cases, there is one with facts relevantly identical to those before the judge, then the decision has already been made. The precedent determines the decision of the instant case. However, the question always at large is, what is the appropriate criterion of relevant identity between the case at issue and any one of those already decided? As no two cases are identical in all respects, answering this question is an inescapable judicial responsibility, and a prerequisite to a claim of precedential control.

The judge's selection of the applicable criterion of relevant identity does not automatically determine which prior decisions are of precedential import. Just as cases may be similar or distinct according to the criterion of similarity used, so also it is with facts, the elemental components of cases. Although the present judge has no power to change the facts of a prior case, she can decide which of those facts are now important and in what way.[13] Whether a given fact is of significance to a decision or irrelevant will depend upon the criterion of similarity in use.[14] The choice of criterion rests upon the present judge, and, although no doubt influenced by the *dicta* of earlier opinions, this choice is not controlled by precedent. The importance of facts to prior decisions as well as to the present decision is thus determined by the present judge.

This is the second respect in which the common law decision is underdetermined by precedent and facts. Not only must the judge choose a criterion of similarity in order to determine the applicability of prior decisions, she mnst also use that criterion to determine the significance of facts within both those prior decisions and the case to be decided. A formal analysis must account for both indeterminacies in the system of common law.

In the following the variable "$c$" ranges over cases, or, more precisely, the facts of cases as determined by courts; "$c_i$" thus picks out some particular set of facts of a case. The notation "$L$" is used for the set of legal predicates, "$L_i$" for a particular one of them.[15] Schematically then the intrinsically legal question facing the judge is of the form "Is $c_i$ an $L_j$?" where $c_i$ is the set of facts comprising the case to be decided and $L_j$ is the legal predicate.

The set of cases in which it was decided that a given predicate, $L_i$, does apply shall be called the "precedent set of $L_i$," and for convenience the set of cases in which it was decided that $L_i$ does not apply shall be called the complement of the precedent set.[16] Let "$P$" designate the set of all cases already

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13. For a striking example of a judge's rearranging of facts, even within the same case, see Justice Rehnquist's opinion in Paul v. Davis, 424 U.S. 693, 694-97 (1976); *cf.* Davis v. Paul, 505 F.2d 1180, 1181 (6th Cir. 1974).

14. This argument is set forth in more detail in Sinclair, *supra* note 6, at 393-95.

15. The set of legal predicates is hierarchically ordered. For example, battery is a tort, and both "tort" and "battery" are members of L—but this is of no special consequence at this stage. Legal predicates may be specified by a list but that list would, of course, not be closed.

16. Obviously it is not a true complement. Also, neither the precedent set nor its complement is necessarily consistent.

decided. It is divided into subsets (not necessarily exclusive) according to the predicates under which its members were decided; let "$P_{Li}$" designate the precedent set of $L_i$ and "$\underline{P}_{Li}$" its complement. Let "$J$" designate the set of courts and "$J_i$" the set of courts of status $i$ (trial, appellate, . . . according to the system). The set of decided cases $P$, and the precedent sets and com- plements, $P_{Li}$ and $\underline{P}_{Li}$, are divided into subsets according to the courts that decided those cases.[17]

The notation "$R$" shall be used for the precedent relation. It is the relation that obtains between two cases, one of which must precede the other in time and in some way have influence over the decision of the other. A common law system, such as is in operation in some areas of the law in most of the states of the United States, is a 4-tuple, $< L, J, P, R >$ where $L, J, P,$ and R are as set forth above.

The next step is to set up the analysis in such a way as to provide an empirically adequate explication of the nature of the precedent relation, $R$. The analysis that follows is along lines suggested by David Lewis' counterpart theory.[18] Qualms about quantifying over possible worlds are not relevant here where only cases are involved. Some scruples might be bothered by hypothetical and future cases and, more seriously, by quantifying over the kinds of things here called facts (the propositions accepted as correct by the court).[19] As to the existence of these "facts," discourse within common law systems simply does carry ontological commitment to representations of facts as fundamental elements of all $c_i$ (most notably all $c_i$ in $P_{Li} \cup \underline{P}_{Li}$). However, the reader might perfer to construct facts out of individuals and events using the methods of categorial grammars.[20] Using counterpart theory as a model also makes it possible to avoid using modal operators, a desirable feature if one is looking toward the possibility of mechanized analysis.

The variables $x, y, z, . . . ,$ range over the particular facts the conjunctions

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17. Standard logic and set theory notations are used throughout. Thus: '$\cup$' for set union (*i.e.*, "$P \cup Q$" means "set $P$, set $Q$, or both"); '$<$' and '$>$' surround ordered sequences; '$\sim$' is the truth functional negation of propositional logic and can be read "it is not the case that . . ."; '.' is the truth functional conjunetion of propositional logic; '$v$' is the truth functional disjunction of propositional logic; '$\supset$' is the truth functional conditional of propositional logic; '$\equiv$' is the truth functional bi-conditional of propositional logic; '$(x)$', the universal quantifier, can be read "for all x . . . "; '$\exists$ x', the existential quantifier, can be read "for some x . . ." or "there is an x such that . . ."; '$=$' is an abbreviation for "is identical to" and '$\neq$' is an abbreviation for "is not identical to."

18. Lewis, *Counterpart Theory and Quantified Modal Logic*, 65 J. PHIL. 113 (1968).

19. A Meinongian analysis along the lines of Parsons, *A Prolegomenon to Meinongian Semantics*, 71 J. PHIL. 561, 561-79 (1974), might be useful in drawing the full range of distinctions needed. On this view a "case" would be, roughly, any set of facts (any proposition) and would thus allow hypothetical and future cases to be considered. The concept would be detailed in much the same way as a Meinongian object. Then actual cases that got before courts would be called something else, say "suits." To every suit there would be a correlate case. While this avenue appears promising, the author has yet to pursue it further.

20. *See* M. CRESSWELL, *supra* note 2; R. MONTAGUE, *supra* note 1.

of which comprise cases. "$x \in c_i$" is an abbreviation for "$x$ *is a fact in* $c_i$;"
"$Sxy$" is an abbreviation for "$x$ is counterpart of $y$;" "$c_i I_k c_j$" is an abbre-
viation for "$c_i$ is relevantly identical (under the criterion $I_k$) to $c_j$."

To use a case in the precedent set it is first necessary to establish some
relationship between that case and the one to be decided. The concept of
"counterpart" is intended to capture this relation between the elements of
the cases. However, the relationship between elements of cases, like the
relationship between cases, is to be determined by the judge in the case to
be decided. It is not given deterministically in the holdings or the *dicta* of
prior cases. Accordingly the counterpart relation is variable according to
purpose and the criteria adopted by the court.[21]

The notation "$I$" has been introduced for criteria of relevant identity and
"$I_k$" for a particular criterion. Relevant identity can be seen either as de-
termining the sum of the counterpart relations, or as the set of principles
or general moral precepts under which the counterpart relations can be
subsumed.[22]

The analysis proceeds by setting forth the properties of the counterpart
relation, $S$, and then building relevant identity, $I_k$, out of it. These basic
notions are then used to construct a model of *stare decisis* using the precedent
relation, $R$.

1.
$$\sim (x)(\exists y)(\exists c_j)(y \in c_i \,.\, Syx)$$
There does not have to be counterpart for every fact.

2.
$$\sim (x)(c_i)(c_j) \,((x \in c_i \,.\, x \in c_j) \supset (c_i = c_j))$$
It is not necessarily true that a fact cannot be in more than one case.

3.
$$(x)(y)(Sxy \supset (\exists c_i)(x \in c_i))$$
Counterparts can be found only in cases.

4.
$$(x)(y)(Sxy \supset (\exists c_i)(y \in c_i))$$
Whatever has a counterpart is in a case.

5.
$$(x)(y)(c_i)((x \in c_i \,.\, y \in c_i \,.\, (\exists z)(Sxz \,.\, Syz)) \supset (x = y))$$
A fact has at most one counterpart in a given case.

6.
$$(x)(c_i)(x \in c_i \supset Sxx)$$
In a case any fact is its own counterpart - S is reflexive.

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21. If necessary "$S$" could be subscripted—$S_i, S_j, \ldots$—to distinguish different choices that
may be made.
22. *See* Sinclair, *supra* note 6, at 393-95.

7.
$$\sim (x)(c_i)(c_j)(x \epsilon c_i \supset (\exists y)(y \epsilon c_j \, . \, Syx))$$

For any two cases there is not necessarily a counterpart in one of any element in the other.[23]

8.
$$\sim (x)(y)(c_i)(c_j)((x \epsilon c_i \, . \, y \epsilon c_j \, . \, Sxy \, . \, i \neq j) \supset$$
$$(\exists w)(\exists z)(w \epsilon c_i \, . \, z \epsilon c_j \, . \, w \neq x \, . \, z \neq y \, . \, Swz))$$

Just because one fact in $c_i$ is counterpart of one fact in $c_j$ it does not necessarily follow that any of the other facts of $c_i$ are counterparts of any of the other facts of $c_j$.

9.
$$(c_i)(c_j)(((x)(\exists y)(x \epsilon c_i \, . \, y \epsilon c_j \, . \, Syx) \, . \, (y)(\exists x)(x \epsilon c_i \, . \, y \epsilon c_j \, . \, Sxy)) \supset (c_i I_k c_j))$$

Two cases are relevantly identical if their elements can be matched as counterparts. This expresses the idea of relevant identity as the sum of the counterpart relations.

It has been noted above that counterpart relations are not fixed and invariate, but rather, are determined by the judge in a particular case as consequences of her choice of criterion of relevant identity. The deduction of particular counterpart relations from the chosen criterion of relevant identity, consistent with the general social and moral principles supporting that choice, is the province of legal argument. It is essentially the question of how these particular facts bear upon the principle in question. It is thus not necessarily truth functional.[24]

As a result two cases might be relevantly identical when only one fact in one case stands arguably in a counterpart relation to a fact in the other case for any *prima facie* significant exogenous purpose. In such a situation all the other facts are arbitrarily matched up as *"ceteris paribus* counterparts." This situation is the clearest illustration of the normal conceptual priority of relevant identity over the counterpart relation. It might appear that this requirement would be better expressed with a filter (a "most if not all" quantifier) rather than the universal quantifier, but this would be incorrect. Under any given social or moral principle as criterion of identity, indefinitely many facts are going to be of no differential consequence whatever, and thus *ceteris paribus* expresses not neglect but the appropriate counterpart relation.[25] It follows therefore that:

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23. *But see* Shelley v. Kraemer, 334 U.S. 1 (1948). To the extent that it found state action in the requirement of resort to courts for ultimate enforcement, it provides a counter example to postulate (7).

24. Wellman, *Practical Reasoning and Judicial Justification: Toward an Adequate Theory*, 57 U. COLO. L. REV. 45, 45-115 (1985).

25. *See* Sinclair, *supra* note 6, at 393-95. That argument suggests that postulate 10 is the only acceptable form.

10.
$$(c_i)(c_j)(c_i I_k c_j \supset ((x)(\exists y)(x \epsilon c_i . y \epsilon c_j . Syx) . (y)(\exists x)(x \epsilon c_i . y \epsilon c_j . Sxy)))$$

Clearly postulates (9) and (10) can be combined in the form:

11.
$$(c_i)(c_j)(((x)(\exists y)(x \epsilon c_i . y \epsilon c_j . Syx) . (y)(\exists x)(x \epsilon c_i . y \epsilon c_j . Sxy)) \equiv$$
$$c_i I_k c_j)$$

Postulate 11—and thus also the appropriateness of 9 and/or 10—should be contentious. *Res judicata* is derivable from postulate 6 and either postulate 9 or postulate 11 where $i=j$. This can be expressed as the theorem:
$$(c_i)(c_i I_x c_i).$$

12.
$$(c_i)(c_j)(c_k)((c_i I_m c_j . c_j I_m c_k) \supset c_i I_m c_k)$$

Transitivity of relevant identity. For this to hold it is necessary to look at the three cases under only one criterion of similarity. If, however, the criteria of similarity are different—say the criterion between $c_j$ and $c_k$ is the one used in $c_j$, but not the one being used in $c_i$, the case under consideration—then clearly transitivity will not necessarily obtain. Thus:

13.
$$\sim (c_i)(c_j)(c_k)((m \neq n) \supset ((c_i I_m c_j . c_j I_n c_k) \supset c_i I_i c_k))$$

14.
$$(x)(y)(z)(Sxy . Syz \supset Sxz)$$

Transitivity of the counterpart relation. Again this will hold only for a particular counterpart relation, that is, only under a particular criterion of relevant identity. It is similar in this way to postulates 12 and 13. Difference in counterpart relation, however, does not follow automatically from difference in criterion of similarity.

There remains the explication of $R$, the precedent relation, in terms of the concept of relevant identity developed above. Clearly $R$ is a relation between cases, typically one being decided under a given predicate and one in the precedent set of that predicate or its complement. $R$ is also not a single relation but a complex of a range of relations, from binding through persuasive precedent. It shall be assumed for the sake of simplicity that there are three strengths of precedential power. Numerical subscripts indicate these different strengths.

For a given case $c_i$ under a given predicate $L_j$ before a given court $J_k$ there will be three accesibility relations[26] with $P_{Lj} \cup \underline{P}_{Lj}$. These relations are functions because the sets of cases picked out by them are unique.

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26. Kripke, *Semantical Considerations on Modal Logic*, 16 ACTA PHILOSOPHICA FENNICA 83 (1963). This model quantifies over cases (as world theory does over possible worlds) and thus avoids the need for modal operators. Accessibility is thus a relation among cases and will facilitate standard analyses of subjunctives such as: "Had *this* been decided then . . ."; "If $c_j$ had been decided thus-and-so, then . . . ."

$R_i$: (binding precendent) for a given court $J_i$, case $c_j$, and predicate $L_i$, $R_i$ picks out the set of cases in $P$ such that if any $c_j$ in that set is relevantly identical to $c_i$ (i.e., if $c_j I_k c_i$, where $I_k$ has been determined to be the criterion of relevant identity) then $J_i$ *must* follow the decision made for $c_j$. Since $R_i$ is a function, $R_i$ $(< J_i, c_i, L_i >)$ is the subset of $P_{Li} \cup \underline{P}_{Li}$ decided by courts *higher* in the hierarchy than $J_i$.

$R_2$: (strongly persuasive precedent) the same as for $R_i$ but with "must have good reason not to" for "*must*" and "of equal status" for "*higher*."

$R_3$: (persuasive precedent) the same as for $R_i$ but with "should take account of" for "*must*" and "of lower status" for "*higher*."

$R$: is the set of all precedential relations.[27]
It follows that *stare decisis*, the doctrine of precedent, will be expressed by:
15.
$$(c_i)(c_j)((c_i I_k c_j \cdot c_j \epsilon R(< J_i, c_i L_i >)) \supset (L_i c_i \equiv (L_i c_i \equiv L_i c_j))[28]$$

The precedent relation, $R$, depends upon the judge's having chosen an *p* appropriate criterion of relevant identity and having used that criterion to order and relate the facts comprising the case before her.[29] These determinations are based upon information and reasoning exogenous to the common law itself. How the judge decides is thus not determined by the common law itself. Ethics, economics, social science, and political considerations are of continuing and unavoidable relevance to the common law decision.[30] It is clear, even in this most formal analysis, that the force of the doctrine of precedent is, as H.L.A. Hart saw,[31] coextensive with the foree of justice.

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27. It might be noted that this is exactly as in the analysis of a discourse in common language. The triple $<J_i, c_i, L_i>$ is the base world and $R_i$, $R_2$, and $R_3$, together form a complex accessibility relation. The properties of this accessibility relation are derived empirically from the nature of the discourse in both the analysis of common language discourse and this analysis of common law. What's more, being empirical phenomena they sometimes go awry. This happens, for example, when there are two cases in a precedent set that are apparently relevantly identical but which have been decided differently. The extraordinary contortions a court or law professor will go through to "reconcile" them are familiar. Rather than showing something wrong with this theory, this is in fact quite strong evidence for it. It is hardly surprising that courts using different criteria of relevant identity should come to different conclusions on facts indistinguishable on either or on a third criterion.

28. Notice that where $c_i = c_j$ it can*not* be deduced from postulate 15 and the *res judicata* theorem, $(c_j)(c_j I_k c_j)$, that: $c_i \epsilon R(<J_i, c_i, L_i>)$. (Such a tempting inference would rest on the confirmation of the consequent, a standard fallacy.) Intuitively, the reason is that a case yet to be decided is not precedent for itself or for any other case and is thus not in the range of any precedent relation. The doctrine of precedent cannot therefore be expressed with the biconditional rather than the conditional. This is an altogether expectable result.

29. A judge who is firmly convinced that a relevantly identical precedent was decided wrongly might simply refuse to follow it. That is not to deny the power of precedent, so it would be wrong to account for it in a model of common law.

30. In those areas of law and during those periods in which there is substantial stability of the relevant social and moral principles and of the relation of classes of facts to them, this kind of analysis could yield a stable, machine processable, decision procedure. However the usefulness of such a device would be continually in doubt: has something changed in the real world to upset that essential stability? If so, how could the device tell?

31. H.L.A. HART, *supra* note 12, at 155-56.

Although the analytic system used could lend itself to mechanization, its application shows two levels of uncertainty: (i) the appropriate criterion of similarity between cases; and (ii) the ordering and hence the relative significance of the facts of the case and precedent cases in question. So the judicial decision is always underdetermined by the facts of the case and the precedents no matter how extensive they may be.

An empirical system can cope with one uncertainty and still be deterministic, but these two layers of uncertainty make the indeterminacy of the common law decision systematic. The problem is analogous to that of the intrinsic indeterminacy of the maxim "aim at the greatest good for the greatest number." One can maximize the good for a given number or maximize the number who get a given good, but not both.[32] The judge cannot, without an exercise of judgment, determine both the impact of precedent and the relevance of the facts.

The most interesting result of this exercise in formal model building is that it makes clear that this double uncertainty is inescapable. A judge must, unavoidably, make a decision, and these same intrinsic indeterminacies that make judicial deciding unavoidable also cast doubt on the possibility of a useful mechanization of common law.[33]

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32. Technically it is described as having two maximands and only one degree of freedom. The problem is easily seen in the instruction: stand outside the law school and pick out the greatest number of tallest students.

33. *Contra* Stone-de Montpensier, *The Compleat Wrangler,* 50 MINN. L. REV. 1001, 1001-25 (1966); *Logic and Law: The Precedence of Precedents,* 51 MINN. L. REV. 655, 655-74 (1966-67). Stone-de Montpensier argued that the common law is comprised of a set of rules deducible from an axiomatic basis and thus could be formalized and deterministic. He also argued, however, that the system would necessarily be incomplete, citing Godel (although he did not explain why we should think Godel's theorem relevant to such a system).